Edexcel Maths C3

Topic Questions from Papers

Functions

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3. The function f is defined by

$$f : x \to \frac{5x+1}{x^2+x-2} - \frac{3}{x+2}, \quad x > 1.$$

(a) Show that $f(x) = \dfrac{2}{x-1}, \quad x > 1.$

(4)

(b) Find $f^{-1}(x)$.

(3)

The function g is defined by

$$g : x \to x^2 + 5, \quad x \in \mathbb{R}.$$

(c) Solve $fg(x) = \frac{1}{4}$.

(3)

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8. The functions f and g are defined by

$$f : x \to 2x + \ln 2, \quad x \in \mathbb{R},$$

$$g : x \to e^{2x}, \quad x \in \mathbb{R}.$$

(a) Prove that the composite function gf is

$$gf : x \to 4e^{4x}, \quad x \in \mathbb{R}.$$

(4)

(b) In the space provided on page 19, sketch the curve with equation $y = \text{gf}(x)$, and show the coordinates of the point where the curve cuts the y-axis.

(1)

(c) Write down the range of gf.

(1)

(d) Find the value of x for which $\frac{d}{dx}[\text{gf}(x)] = 3$, giving your answer to 3 significant figures.

(4)

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Question 8 continued

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7. For the constant k, where $k > 1$, the functions f and g are defined by

$$\text{f}: x \mapsto \ln(x + k), \quad x > -k,$$
$$\text{g}: x \mapsto |2x - k|, \quad x \in \mathbb{R}.$$

(a) On separate axes, sketch the graph of f and the graph of g.

On each sketch state, in terms of k, the coordinates of points where the graph meets the coordinate axes.

(5)

(b) Write down the range of f.

(1)

(c) Find $\text{fg}\left(\frac{k}{4}\right)$ in terms of k, giving your answer in its simplest form.

(2)

The curve C has equation $y = \text{f}(x)$. The tangent to C at the point with x-coordinate 3 is parallel to the line with equation $9y = 2x + 1$.

(d) Find the value of k.

(4)

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Question 7 continued

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6. The function f is defined by

$$f : x \mapsto \ln(4 - 2x), \quad x < 2 \quad \text{and} \quad x \in \mathbb{R}.$$

(a) Show that the inverse function of f is defined by

$$f^{-1} : x \mapsto 2 - \frac{1}{2}e^x$$

and write down the domain of f^{-1}.

(4)

(b) Write down the range of f^{-1}.

(1)

(c) In the space provided on page 16, sketch the graph of $y = f^{-1}(x)$. State the coordinates of the points of intersection with the x and y axes.

(4)

The graph of $y = x + 2$ crosses the graph of $y = f^{-1}(x)$ at $x = k$.

The iterative formula

$$x_{n+1} = -\frac{1}{2}e^{x_n}, \quad x_0 = -0.3$$

is used to find an approximate value for k.

(d) Calculate the values of x_1 and x_2, giving your answers to 4 decimal places.

(2)

(e) Find the value of k to 3 decimal places.

(2)

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Question 6 continued

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5. The functions f and g are defined by

$$\mathrm{f} : x \mapsto \ln(2x-1), \qquad x \in \mathbb{R},\ x > \frac{1}{2},$$

$$\mathrm{g} : x \mapsto \frac{2}{x-3}, \qquad x \in \mathbb{R},\ x \neq 3.$$

(a) Find the exact value of fg(4).

(2)

(b) Find the inverse function $\mathrm{f}^{-1}(x)$, stating its domain.

(4)

(c) Sketch the graph of $y = |\mathrm{g}(x)|$. Indicate clearly the equation of the vertical asymptote and the coordinates of the point at which the graph crosses the y-axis.

(3)

(d) Find the exact values of x for which $\left|\frac{2}{x-3}\right| = 3$.

(3)

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Question 5 continued

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8. The functions f and g are defined by

$$f : x \mapsto 1 - 2x^3, \quad x \in \mathbb{R}$$

$$g : x \mapsto \frac{3}{x} - 4, \quad x > 0, \quad x \in \mathbb{R}$$

(a) Find the inverse function f^{-1}.

(2)

(b) Show that the composite function gf is

$$gf : x \mapsto \frac{8x^3 - 1}{1 - 2x^3}.$$

(4)

(c) Solve $gf(x) = 0$.

(2)

(d) Use calculus to find the coordinates of the stationary point on the graph of $y = gf(x)$.

(5)

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Question 8 continued

Q8

(Total 13 marks)

TOTAL FOR PAPER: 75 MARKS

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4. The function f is defined by

$$\text{f} : x \mapsto \frac{2(x-1)}{x^2-2x-3} - \frac{1}{x-3}, \quad x > 3.$$

(a) Show that $\text{f}(x) = \dfrac{1}{x+1}, \quad x > 3.$

(4)

(b) Find the range of f.

(2)

(c) Find $\text{f}^{-1}(x)$. State the domain of this inverse function.

(3)

The function g is defined by

$$\text{g} : x \mapsto 2x^2 - 3, \quad x \in \mathbb{R}.$$

(d) Solve $\text{fg}(x) = \dfrac{1}{8}$.

(3)

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Question 4 continued

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5. The functions f and g are defined by

$$f : x \mapsto 3x + \ln x, \quad x > 0, \quad x \in \mathbb{R}$$

$$g : x \mapsto e^{x^2}, \quad x \in \mathbb{R}$$

(a) Write down the range of g.

(1)

(b) Show that the composite function fg is defined by

$$fg : x \mapsto x^2 + 3e^{x^2}, \quad x \in \mathbb{R}.$$

(2)

(c) Write down the range of fg.

(1)

(d) Solve the equation $\frac{d}{dx}\left[fg(x)\right] = x(xe^{x^2} + 2)$.

(6)

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Question 5 continued

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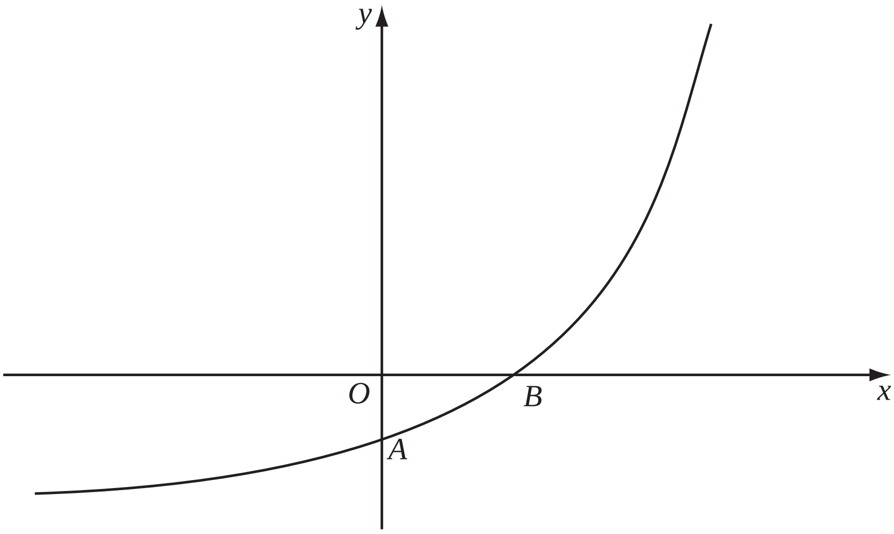

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$, $x \in \mathbb{R}$.

The curve meets the coordinate axes at the points $A(0, 1-k)$ and $B(\frac{1}{2}\ln k, 0)$, where k is a constant and $k > 1$, as shown in Figure 2.

On separate diagrams, sketch the curve with equation

(a) $y = |\mathrm{f}(x)|$,
(3)

(b) $y = \mathrm{f}^{-1}(x)$.
(2)

Show on each sketch the coordinates, in terms of k, of each point at which the curve meets or cuts the axes.

Given that $\mathrm{f}(x) = \mathrm{e}^{2x} - k$,

(c) state the range of f ,
(1)

(d) find $\mathrm{f}^{-1}(x)$,
(3)

(e) write down the domain of f^{-1}.
(1)

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Question 5 continued

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7. The function f is defined by

$$f(x) = 1 - \frac{2}{(x+4)} + \frac{x-8}{(x-2)(x+4)}, \quad x \in \mathbb{R}, \ x \neq -4, \ x \neq 2$$

(a) Show that $f(x) = \dfrac{x-3}{x-2}$ **(5)**

The function g is defined by

$$g(x) = \frac{e^x - 3}{e^x - 2}, \quad x \in \mathbb{R}, \ x \neq \ln 2$$

(b) Differentiate $g(x)$ to show that $g'(x) = \dfrac{e^x}{(e^x - 2)^2}$ **(3)**

(c) Find the exact values of x for which $g'(x) = 1$ **(4)**

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Question 7 continued

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8. (a) Write down $\sin 2x$ in terms of $\sin x$ and $\cos x$.

(1)

(b) Find, for $0 < x < \pi$, all the solutions of the equation

$$\operatorname{cosec} x - 8\cos x = 0$$

giving your answers to 2 decimal places.

(5)

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6. The function f is defined by

$$f: x \mapsto \frac{3-2x}{x-5}, \quad x \in \mathbb{R}, \ x \neq 5$$

(a) Find $f^{-1}(x)$.

(3)

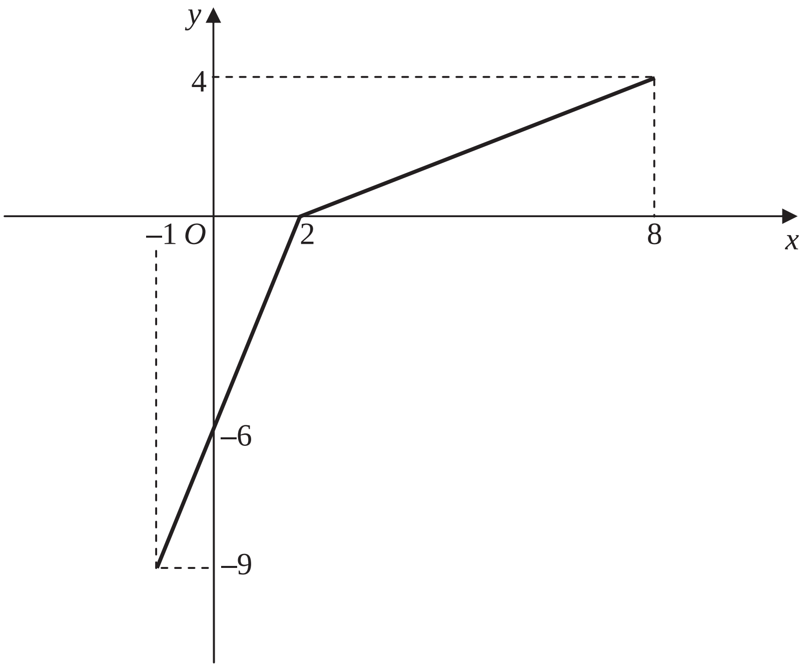

Figure 2

The function g has domain $-1 \leqslant x \leqslant 8$, and is linear from (–1, –9) to (2, 0) and from (2, 0) to (8, 4). Figure 2 shows a sketch of the graph of $y = g(x)$.

(b) Write down the range of g.

(1)

(c) Find gg(2).

(2)

(d) Find fg(8).

(2)

(e) On separate diagrams, sketch the graph with equation

(i) $y = |g(x)|$,

(ii) $y = g^{-1}(x)$.

Show on each sketch the coordinates of each point at which the graph meets or cuts the axes.

(4)

(f) State the domain of the inverse function g^{-1}.

(1)

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Question 6 continued

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4. The function f is defined by

$$f : x \mapsto 4 - \ln(x+2), \quad x \in \mathbb{R}, \ x \geqslant -1$$

(a) Find $f^{-1}(x)$.

(3)

(b) Find the domain of f^{-1}.

(1)

The function g is defined by

$$g : x \mapsto e^{x^2} - 2, \quad x \in \mathbb{R}$$

(c) Find $fg(x)$, giving your answer in its simplest form.

(3)

(d) Find the range of fg.

(1)

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7. The function f is defined by

$$\mathrm{f} : x \mapsto \frac{3(x+1)}{2x^2+7x-4} - \frac{1}{x+4}, \qquad x \in \mathbb{R}, x > \frac{1}{2}$$

(a) Show that $\mathrm{f}(x) = \dfrac{1}{2x-1}$ **(4)**

(b) Find $\mathrm{f}^{-1}(x)$ **(3)**

(c) Find the domain of f^{-1} **(1)**

$$\mathrm{g}(x) = \ln(x+1)$$

(d) Find the solution of $\mathrm{fg}(x) = \dfrac{1}{7}$, giving your answer in terms of e. **(4)**

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Question 7 continued

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6. The functions f and g are defined by

$$f : x \mapsto e^x + 2, \qquad x \in \mathbb{R}$$

$$g : x \mapsto \ln x, \qquad x > 0$$

(a) State the range of f.

(1)

(b) Find $fg(x)$, giving your answer in its simplest form.

(2)

(c) Find the exact value of x for which $f(2x+3) = 6$

(4)

(d) Find f^{-1}, the inverse function of f, stating its domain.

(3)

(e) On the same axes sketch the curves with equation $y = f(x)$ and $y = f^{-1}(x)$, giving the coordinates of all the points where the curves cross the axes.

(4)

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Question 6 continued

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7. $$h(x) = \frac{2}{x+2} + \frac{4}{x^2+5} - \frac{18}{(x^2+5)(x+2)}, \qquad x \geqslant 0$$

(a) Show that $h(x) = \dfrac{2x}{x^2+5}$ **(4)**

(b) Hence, or otherwise, find $h'(x)$ in its simplest form. **(3)**

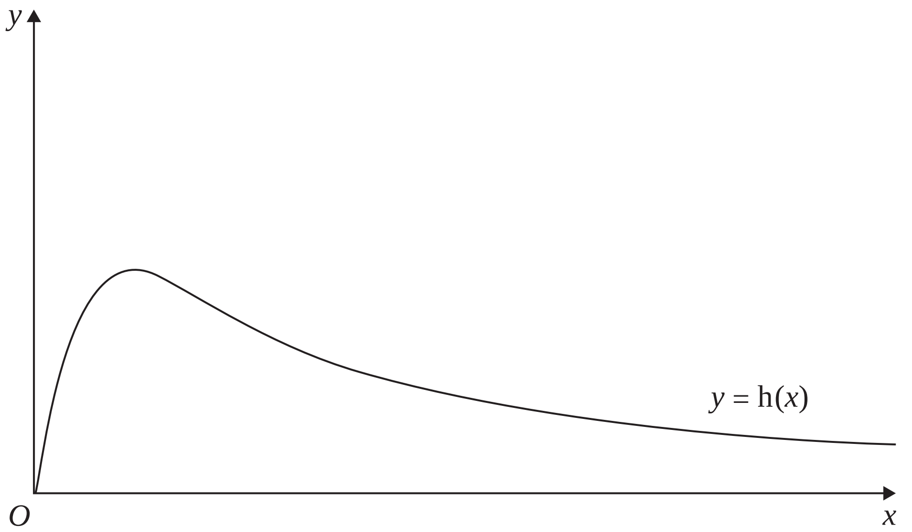

Figure 2

Figure 2 shows a graph of the curve with equation $y = h(x)$.

(c) Calculate the range of $h(x)$. **(5)**

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Question 7 continued

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4. The functions f and g are defined by

$$f : x \mapsto 2|x| + 3, \qquad x \in \mathbb{R},$$

$$g : x \mapsto 3 - 4x, \qquad x \in \mathbb{R}$$

(a) State the range of f.

(2)

(b) Find fg(1).

(2)

(c) Find g^{-1}, the inverse function of g.

(2)

(d) Solve the equation

$$gg(x) + [g(x)]^2 = 0$$

(5)

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Question 4 continued

P 4 2 9 5 3 A 0 1 1 2 8

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7. The function f has domain $-2 \leqslant x \leqslant 6$ and is linear from $(-2, 10)$ to $(2, 0)$ and from $(2, 0)$ to $(6, 4)$. A sketch of the graph of $y = \mathrm{f}(x)$ is shown in Figure 1.

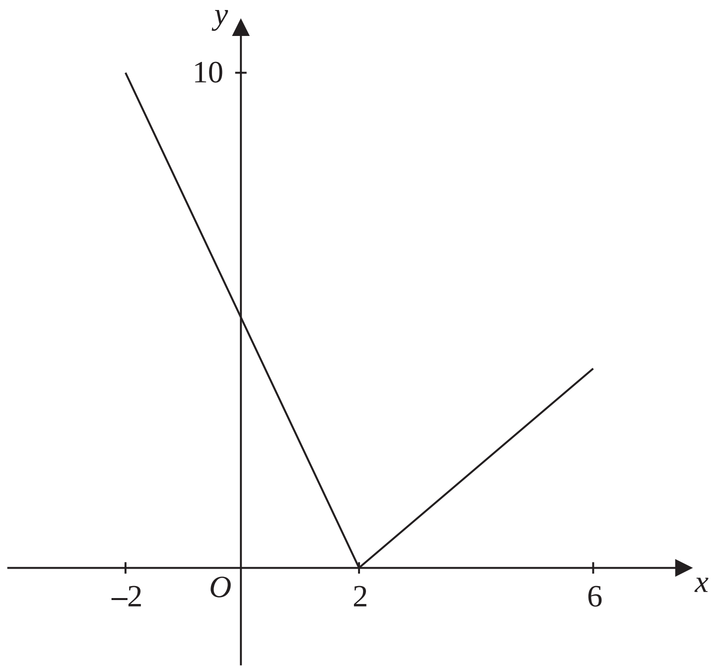

Figure 1

(a) Write down the range of f. **(1)**

(b) Find ff(0). **(2)**

The function g is defined by

$$\mathrm{g} : x \rightarrow \frac{4 + 3x}{5 - x}, \qquad x \in \mathbb{R}, \qquad x \neq 5$$

(c) Find $\mathrm{g}^{-1}(x)$ **(3)**

(d) Solve the equation $\mathrm{gf}(x) = 16$ **(5)**

P 4 3 0 1 6 A 0 2 4 3 2

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Question 7 continued

P 4 3 0 1 6 A 0 2 5 3 2

Turn over

Core Mathematics C3

Candidates sitting C3 may also require those formulae listed under Core Mathematics C1 and C2.

Logarithms and exponentials

$$e^{x \ln a} = a^x$$

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad (A \pm B \neq (k + \tfrac{1}{2})\pi)$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

Differentiation

f(x)	**f′(x)**
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\frac{f(x)}{g(x)}$	$\frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$

Core Mathematics C2

Candidates sitting C2 may also require those formulae listed under Core Mathematics C1.

Cosine rule

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Binomial series

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$$

where $\binom{n}{r} = {}^n\mathrm{C}_r = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1\times 2}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{1\times 2\times \ldots \times r}x^r + \ldots \qquad (|x| < 1, n \in \mathbb{R})$$

Logarithms and exponentials

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Geometric series

$$u_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Numerical integration

The trapezium rule: $\int_a^b y \, dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

Core Mathematics C1

Mensuration

Surface area of sphere = $4\pi r^2$

Area of curved surface of cone = $\pi r \times$ slant height

Arithmetic series

$u_n = a + (n - 1)d$

$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n[2a + (n - 1)d]$